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DEPARTMENT OF ECONOMICS

RESEARCH MEMORANDUM

THE LONG TERM ELASTICITY OF THE MILK SUPPLY WITH RESPECT TO THE MILK PRICE IN THE NETHERLANDS IN THE PERIOD 1969-1984

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Abstract

This paper deals with the calculation of the long term elasticity of the milk supply with respect to the milk price in the Netherlands during the period 1969-1984. Because the individual farmer decides upon (changes in) this supply, we consider the underlying decision process at the farm as an obvious point of departure. For this farm we formulate a dynamic, alphanumerically specified model of supply. From this model we derive decision rules for the optimal level of the (des)investments in the dairy cow stock. The resulting reaction equations specify these levels as a function of, among other things, present and expected prices. So these relations provide a starting point for e.g. the estimation of this long run elasticity.

1. Introduction

In research concerning the effect of the producer's price for milk on the milk supply often use is made of the supply model proposed by Nerlove, compare for instance [1]. In its most simple formulation this model specifies the supply of an agricultural product in a period as a linear function of the price that the producers expect for that period.

In this paper, which concerns the estimation of the long term elasticity of the milk supply with respect to the milk price in the Netherlands during the period from roughly the start of the common dairy market till the introduction of the super levy, use is made of a variant of the Nerlove model, too. However, contrary to the usual approach, we do not postulate this model, but we derive it from an optimization model. In such a way an economic underpinning of the supply specification to be used in the estimation phase is attained.

The point of departure here is a farm that is primarily directed towards milk production by cows from own breeding. In every year within the decision horizon such a farm has to take in reaction on the changing circumstances, amongst them the level of the milk price, decisions, as to how the farm will be run in that particular year. Furthermore, decisions have to be made annually on the direction and the volume of investment in live and dead stock and on whether these investments should be financed by own or borrowed funds. For such a firm we develop a model of the determinants of the milk production in the long run, that means at a term of more than one year. Because this production is equal to the product of the average milk yield per cow and the number of lactating cows, a change in the volume of this production can be realized by this average yield, via the number of lactating animals or via a combination of these two possibilities. In this paper we take the yield development as autonomous, so the size (and composition by age) of the dairy cow stock determines the development of the milk production, other factors left aside.

Although decisions with respect to the live stock must always fit in the possibilities qua labour, dead stock and capital that the farm can dispose of, these factors will not be considered here. We suppose that the capacity restrictions caused by these factors are not binding in any period. As a result of this assumption the derivation of the supply specification does not come up for discussion in its full generality. However, by this simplification the exposition of the method followed for the derivation gains clearness.

In section 2 the problem which the farmer faces every year, is briefly sketched. The essense of this problem, given our assumptions, is the determination of the optimal size and age composition of the different live stock categories and the corresponding levels of in- and outflow in these categories. As a criterion for these decisions the farmer uses the maximization of the value of the (discounted) cash flows caused by his decisions. In section 3 the model of this problem is given. Following an often used supposition in this type of model we assume that the revenues and expenditures can be represented by linear and quadratic functions of the state and decision variables. These elements of the criterion function are alpha-numerically specified, that is in letters and numbers. This by now alpha-numerically specified decision problem is solved in section 4. The solution of this model supplies linear decision rules for among other things the sizes of the inflow in and the outflow out of the dairy cow stock. These relations identify the variables relevant for these decisions - among them present and expected milk prices -, the term at which these variables have impact and the specific significance of each of them. In such a way they supply a starting point for an empirical investigation into the relation between changes in the size of the dairy herd and the level of the (expected) milk price. The modelling of these price expectation is the subject of section 5. Due to the limited number of available data and some problems of a statistical nature, reducing the specifications derived turns out to be unavoidable. After these preparations we proceed to the estimation of the long term elasticity. In section 8, at last, the direction in which this research could be continued, is shortly indicated.

2. The problem

The milk supply of a farm (or a country) during a period is equal to the product of the average yield per cow and the number of animals, on average participating in production during that period. Hence, the dairy farmer can influence the level of supply, in reaction to for instance a change in the milk price, via the average yield, via the number of lactating cows or via a combination of these two possibilities. However, the term on which a change in these factors can be realized and has influence on the level of supply, is different. Measures with respect to the average yield practically without delay result in a change of this average and so in the level of supply. However, a modification of this supply by de- or increasing the dairy cow stock takes generally speaking considerably more time. This is caused by the fact that a structural change in the size of the dairy herd can be achieved via a change in the level of the inflow of heifers. Because heifers must pass through a gestation period, this takes at least nine months. During this period the size of the stock can be modified by the level of culling for economic reasons, coming on top of the outflow for biological reasons. However, under normal circumstances, this type of culling is realized gradually, because of its consequences for the price formation on the beef market.

With $\overline{mgk}$ for the average yield per cow and $\bar{c}$ for the number of animals on average participating in production, the split-up of the milk supply, mp, is expressed by

$$mp = \overline{mgk}.\bar{c} \tag{2.1}$$

The effect of a change in the price of milk, pm, on the average yield and the size of the dairy herd and so on the level of milk supply can be measured by the elasticity of the milk supply with respect to the milk price. This elasticity is defined as the ratio of the (procentual) change in the milk supply and the (procentual) change in milk price:

$$\frac{\dfrac{\Delta mp}{mp}}{\dfrac{\Delta pm}{pm}} = \frac{\dfrac{\Delta mp}{\Delta pm}}{\dfrac{mp}{pm}} \tag{2.2}$$

Using (2.1) this elasticity becomes

$$\frac{\dfrac{\Delta mp}{\Delta pm}}{\dfrac{mp}{pm}} = \frac{\Delta\overline{mgk}}{\Delta pm}.\frac{pm}{\overline{mgk}} + \frac{\Delta\bar{c}}{\Delta pm}.\frac{pm}{\bar{c}} \tag{2.3}$$

Now this paper centers around the estimation of the term $\frac{\Delta\bar{c}}{\Delta pm}\cdot\frac{pm}{\bar{c}}$, the long term elasticity of milk supply with respect to the milk price.

To get an idea of the considerations which determine the size and age composition of the dairy cow stock, and so the levels of in- and outflow, we consider an individual dairy farm that is primarily directed towards milk production by cows from own breeding. In so far as the farmer judges a change of the stock size desirable, he chooses in every period from among the heifer calves, that are born in that period out of his herd, a number for the purpose of breeding. All other heifer calves and all the bull calves he sells for fattening to other specialized farms. As soon as the selected calves have reached the age when they can reproduce, they are put in calf (inseminated), if they still meet the selection requirements, and sold for slaughter if they do not. After completing the gestation period of nine months as heifer in calf, these animals enter the farm's dairy herd as cow. After several lactation periods (and calves) they are finally sold for slaughter, because they are no longer sufficiently productive. For reasons to be explained in the section to come the farmer is neither allowed to buy breeding-cattle from other dairy farms nor to sell it to other dairy farms.

Now, every year again, the farmer faces the same problem. How many of the heifer calves born should be retained at the farm for breeding, how many heifers should be sold for slaughter or put in calf and finally how many cows should be culled. As soon as he has reached his decision, the development of the live stock in that period is known, given the opening stock and ignoring loss by natural death. We assume that the dairy farmer must take such decisions for T consecutive years. At the end of year T he sells his live stock to a new owner.

In deciding upon these questions it holds that the possibilities in a particular period are partly dependent on decisions taken in the past, just as this period's decisions (co)determine the farm's future herd development. It also holds that the farmer in determining the size and age composition of the stock must take into account the capacities of labor, dead stock and funds he has at his disposal. His decisions must always fit within the framework given by these factors. In this paper however we will neither pay attention to the coherence and interaction between these factors and the live stock nor to the possibility and consequences of extending the capacities of these factors. For simplicity's sake we confine ourselves to the live stock. Extensions are dealt with in [2].

3. The model

We assume that the lactation and dry period together make up a year, so every cow in calf gives birth to one calf a year, with equal probability a heifer or a bull calf. During the year following on that in which a heifer calf is born, it enters the heifer (or yearling) category. Heifers can be put in calf (inseminated) or sold for slaughter, either in the year of entering the heifer category or later on. We suppose that between the moment of a heifer's insemination and its calving lies a period of a year, too.

Let vk_t, p_t, v_t, c_t, $t = 0,1,\dots,T$ denote the number of respectively heifer calves, heifers, heifers in calf and lactating cows at the farm at time t, vvk_t, vp_t, vc_t the number of heifer calves, heifers and cows, sold for slaughter in year t, and d_t the number of heifers put in calf in year t. The development of the farm's herd can now be represented by the following equations:

$$\left.\begin{aligned} vk_t &= \tfrac{1}{2}(v_{t-1} + c_{t-1}) - vvk_t \\ p_t &= vk_{t-1} + p_{t-1} - vp_t - d_t \\ v_t &= d_t \\ c_t &= v_{t-1} + c_{t-1} - vc_t \end{aligned}\right\} \quad t = 1,\dots,T \tag{3.1}$$

In matrix notation this reads

$$Y_t = C_1 Y_{t-1} + C_2 X_t, \tag{3.2}$$

where

$$Y_t' = [vk_t, p_t, v_t, c_t], \quad X_t' = [vvk_t, vp_t, d_t, vc_t],$$

the vectors of the state and decision variables respectively, and

$$C_1 = \begin{bmatrix} 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \qquad C_2 = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

As cattle transactions between dairy farms have no influence on the investment level of the sector as a whole, the quantity we are primarily interested in, such transactions will be left out of consideration. In view of that the decision variables are required to be non-negative,

$$X_t \geqq 0 \tag{3.3}$$

Of course, no more heifer calves, heifers or cows can be sold than available,

$$D_1X_t \leqq D_2Y_{t-1}, \quad t = 1,\dots,T \tag{3.4}$$

where

$$D_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad D_2 = \begin{bmatrix} 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

In what follows, we will restrict ourselves to the situation where

$$X_t > 0 \quad \text{and} \quad D_1X_t < D_2Y_{t-1} \quad , \quad t = 1,\dots,T \tag{3.5}$$

so the vector X_t never reaches its minimum or maximum. This restriction is based on the assumption, that in reality these decision variables more often than not will float within the range between these extremes and in but a few cases will assume the extreme value. As a consequence, the decision problem to be formulated at the end of this section is sizably simplified. Also, the opening stock being positive,

$$Y_0 = \bar{Y}_0 \ (> 0), \tag{3.6}$$

the vector of state variables Y_t will always be positive.

One can, on good grounds, hold the view that a farmer, in choosing from a set of alternatives, is satisfied, as soon as he reaches his aspiration level. In this paper, however, we will not proceed from a satisfying, but from a maximizing concept. The objective used here is maximization of the value of (discounted) cash flows, generated by the farmer's decisions. This criterion, though one-sided, without doubt forms an important element in comparing alternatives, directly related as it is to the consumption possibilities of these production/consumption households. Of course, in such an approach leisure has no value.

Revenues accrue to the farm from the delivery of milk to the dairy industry, the sale of heifer and bull calves for fattening, and the sale of heifers and culled cows for slaughter.

The level of milk production by the dairy herd depends on many factors. Important in the long term analysis here are breed, age composition and genetic potential of the average cow. Keeping breed constant we suppose that the revenues from milk in year t are

$$pm_t(1+g)^t\{a_1c_{t-1} + a_3v_{t-1} - a_5vc_t\}, \tag{3.7}$$

where g denotes the genetic improvement in percent a year and a_1, a_3, a_5 the milk yield per dairy cattle category (for culled cows a_1-a_5).

Revenues from the sale of cattle amount to

$$pk_t\{\tfrac{1}{2}\,(c_{t-1} + v_{t-1}) + vvk_t\} + pp_t.vp_t + pc_t.vc_t, \tag{3.8}$$

where pk_t, pp_t and pc_t denote the price of a calf, a heifer and a culled cow respectively. We suppose that these prices are independent of the numbers sold.

In a more complete representation the revenues side also comprises cash receipts from borrowing, but, as remarked before, the aspect of financing the investment/production activities by own or borrowed funds will not be considered in this paper.

Expenditures are done for the acquisition of dead stock, the payment of interest and redemption of debt and for buying concentrates, fertiliser, fuel etc. However, in this paper, we confine ourselves to the expenses for the live stock. We will specify these expenses as a linear-quadratic function of the distinct cattle categories. Having a linear revenues function we achieve in such a manner an optimal size of the live stock to exist. The parameters of this expenditures function reflect the prices of the inputs, such as fodder bought, and the state of technology. We assume, that all of these coefficients change conform inflation during the planning period.

Within the expenditures for the live stock we discern three components. The first of them comprises the expenses determined by the size of the several cattle categories, the second those dependent on the age composition of a category and the third the expenditures not traceable to either size or age composition. The first two components are represented by means of quadratic functions and the third via a linear relation.
Leaving inflation a moment aside the size dependent expenditures consist of the following four components, one for each cattle category,

$$\begin{aligned}
\tfrac{1}{2}\, b_1 vk_t^2 &= \tfrac{1}{2}\, b_1 (\tfrac{1}{2}\, c_{t-1} + \tfrac{1}{2}\, v_{t-1} - vvk_t)^2 \\
\tfrac{1}{2}\, b_2 p_t^2 &= \tfrac{1}{2}\, b_2 (p_{t-1} + vk_{t-1} - vp_t - d_t)^2 \\
\tfrac{1}{2}\, b_3 v_t^2 &= \tfrac{1}{2}\, b_3 d_t^2 \\
\tfrac{1}{2}\, b_4 c_t^2 &= \tfrac{1}{2}\, b_4 (c_{t-1} + v_{t-1} - vc_t)^2
\end{aligned} \tag{3.9}$$

On top of these come the age dependent expenditures arising, when the animals within a category on average become older or younger,

$$\begin{aligned}
&\tfrac{1}{2}\, b_5 (p_{t-1} - vp_t - d_t)^2 \\
&\tfrac{1}{2}\, b_6 (c_{t-1} - vc_t)^2
\end{aligned} \tag{3.10}$$

If p_{t-1} is equal to $vp_t + d_t$, the breeding expenses for heifers in that period amount to $\frac{1}{2}\, b_2 vk_{t-1}^2$. However, if vp_t and d_t are both equal to zero, then these expenses total $\frac{1}{2}\, b_2 (p_{t-1} + vk_{t-1})^2 + \frac{1}{2}\, b_5 p_{t-1}^2$. For the heifer calves and the heifers in calf an age dependent component is not added, because these animals can not stick in their category.

The sum of the expenditures components (3.9) and (3.10) will be denoted by the symbol TE_t.

The remaining part, not traceable to either category size or age composition, is represented by

$$d(\alpha_1 vk_t + \alpha_2 p_t + \alpha_3 v_t + \alpha_4 c_t), \tag{3.11}$$

where the coefficients α_i, $i = 1,\ldots,4$, reduce the four categories to one only.

The net returns to the farmer in guilders of constant purchasing power can now be summarized by the following expression:

$$NR_t = \left[\prod_{j=1}^{t} (1+i_j)\right]^{-1} \{P'_{y,t} Y_{t-1} + P'_{x,t} X_t\} - \frac{1}{2} [Y'_{t-1} X'_t] \begin{bmatrix} A_1 & A_2 \\ A'_2 & A_4 \end{bmatrix} \begin{bmatrix} Y_{t-1} \\ X_t \end{bmatrix}, \quad t = 1,\ldots,T-1 \tag{3.12}$$

where i_j denotes the inflation percentage in year j,

$$P'_{y,t} = \Bigg[-\prod_{j=1}^{t} (1+i_j)\alpha_1 d,\ -\prod_{j=1}^{t} (1+i_j)\alpha_2 d,\ \frac{1}{2} pk_t + pm_t(1+g)^t a_3 +$$

$$- \prod_{j=1}^{t} (1+i_j)\alpha_3 d,\ \frac{1}{2} pk_t + pm_t(1+g)^t a_1 - \prod_{j=1}^{t} (1+i_j)\alpha_4 d\Bigg],$$

$$P'_{x,t} = [pk_t,\ pp_t,\ 0,\ pc_t - pm_t(1+g)^t a_5]$$

and

$$A_1 = \frac{\partial^2 TE_t}{\partial Y^2_{t-1}},\ A_2 = \frac{\partial^2 TE_t}{\partial Y_{t-1} \partial X_t},\ A_4 = \frac{\partial^2 TE_t}{\partial X^2_t}$$

For year T, the sale of the stock comes on top of the revenues.

Now that a specification of net returns is available, the decision problem the farmer faces in the first year within the planning horizon can be represented by the following model, compare also [3],

$$\max F = \sum_{t=1}^{T} \beta^t \left\{ \left[\prod_{j=1}^{t} (1+ {}_1i_j^E) \right]^{-1} \{ {}_1P_{y,t}'^E Y_{t-1} + {}_1P_{x,t}'^E X_t \} + \right.$$

$$\left. - \frac{1}{2} [Y_{t-1}' X_t'] \begin{bmatrix} A_1 & A_2 \\ A_2' & A_4 \end{bmatrix} \begin{bmatrix} Y_{t-1} \\ X_t \end{bmatrix} \right\} + \beta^T \left[\prod_{j=1}^{T} (1 + {}_1i_j^E) \right]^{-1} \{ {}_1P_{y,T+1}'^E Y_T \} \qquad (3.13)$$

subject to

$$Y_t = C_1 Y_{t-1} + C_2 X_t$$

$$Y_0 = \bar{Y}_0$$

Here β denotes the discount factor the farmer uses and ${}_1i_j^E$ the inflation percentage that he in year 1 expects to be valid for year j. The vectors ${}_1P_{y,t}^E$ and ${}_1P_{x,t}^E$ specify his expectations in year 1 with respect to the returns from milk delivery and cattle sales in year t,

$$ {}_1P_{y,t}'^E = \left[- \prod_{j=1}^{t} (1 + {}_1i_j^E)\alpha_1 d,\ - \prod_{j=1}^{t} (1 + {}_1i_j^E)\alpha_2 d,\ \frac{1}{2}\, {}_1pk_t^E + {}_1pm_t^E (1+g)^t a_3 + \right.$$

$$\left. - \prod_{j=1}^{t} (1 + {}_1i_j^E)\alpha_3 d,\ \frac{1}{2}\, {}_1pk_t^E + {}_1pm_t^E (1+g)^t a_1 - \prod_{j=1}^{t} (1 + {}_1i_j^E)\alpha_4 d \right]$$

$$ {}_1P_{x,t}'^E = [{}_1pk_t^E,\ {}_1pp_t^E,\ 0,\ {}_1pc_t^E - {}_1pm_t^E (1+g)^t a_5]$$

Finally, the vector ${}_1P_{y,T+1}'^E = [{}_1pk_{T+1}^E,\ {}_1pp_{T+1}^E,\ \frac{1}{2}({}_1pp_{T+1}^E + {}_1pc_{T+1}^E),\ {}_1pc_{T+1}^E]$ denotes the prices the farmer expects to receive from selling his live stock to a new owner at the end of the planning horizon. The expected prices for the first year are, of course, equal to the actual prices in that year, i.e. ${}_1i_1 = i_1$, ${}_1P_{y,1}^E = P_{y,1}$ and ${}_1P_{x,1}^E = P_{x,1}$.

4. The solution

The decision problem (3.13) (and those for the following years which possess the same structure) can be solved in several ways, e.g. recursively. Now for some as yet unknown reason it holds, that

$$(C_1 - C_2A_4^{-1}A_2')^2 = 0 \tag{4.1}$$

and

$$(C_1 - C_2A_4^{-1}A_2')'(A_1 - A_2A_4^{-1}A_2') = 0 \tag{4.2}$$

Using these features we can obtain the optimal solution in a simple way. This solution is

$$X_t = -A_4^{-1}A_2'Y_{t-1} + Q_t^{-1}W_t, \qquad t = 1,\dots,T-2 \tag{4.3}$$

where

$$Q_t = A_4 + \beta C_2'(A_1 - A_2A_4^{-1}A_2')C_2, \tag{4.4}$$

$$\begin{aligned} W_t = {} & \left[\prod_{j=1}^{t}(1+i_j)\right]^{-1}\{P_{x,t} + (1 + {}_ti^E_{t+1})^{-1}\beta C_2'({}_tP^E_{y,t+1} - A_2A_4^{-1}\,{}_tP^E_{x,t+1}) + \\ & + \{(1 + {}_ti^E_{t+1})(1 + {}_ti^E_{t+2})\}^{-1}\beta^2C_2'(C_1' - A_2A_4^{-1}C_2'). \\ & ({}_tP^E_{y,t+2} - A_2A_4^{-1}\,{}_tP^E_{x,t+2})\} \end{aligned} \tag{4.5}$$

For space considerations the slightly different expressions for the years T-1 and T are omitted. For the same reason we will not write down the whole solution in extenso. Instead we present the decision rules in which we are interested here: the investments, d_t, and the desinvestments, vc_t.

For the simple model considered here, the optimal level of the inflow of heifers in calf in the dairy stock is given by the following expression, obtained by using a formula manipulation language,

$$\begin{aligned} d_t = {} & -\frac{b_4+b_6}{n_3}\,pp_t + \frac{\beta(b_4+b_6)}{(1+{}_ti^E_{t+1})n_3}\,{}_tpk^E_{t+1} + \frac{\beta(1+g)^{t+1}\{b_4(a_3-a_5)+b_6a_3\}}{(1+{}_ti^E_{t+1})n_3}. \\ & {}_tpm^E_{t+1} + \frac{\beta b_4}{(1+{}_ti^E_{t+1})n_3}\,{}_tpc^E_{t+1} + \frac{\beta^2b_6}{(1+{}_ti^E_{t+1})(1+{}_ti^E_{t+2})n_3}\,{}_tpk^E_{t+2} + \end{aligned}$$

$$+ \frac{\beta^2(1+g)^{t+2}b_6(a_1-a_5)}{(1+{}_ti^E_{t+1})(1+{}_ti^E_{t+2})n_3}\,{}_tpm^E_{t+2} + \frac{\beta^2 b_6}{(1+{}_ti^E_{t+1})(1+{}_ti^E_{t+2})n_3}\,{}_tpc^E_{t+2} +$$

$$- \frac{\beta d\{\alpha_3(b_4+b_6) + \beta\alpha_4 b_6\}}{(2\beta b_4 b_6 + b_4 b_3 + b_6 b_3)}\ , \tag{4.6}$$

where $n_3 = \prod_{j=1}^{t}(1+i_j)(2\beta b_4 b_6+b_4 b_3+b_6 b_3)$.

For the optimal culling level we find

$$vc_t = \frac{b_4}{n_4}\,v_{t-1} + c_{t-1} - \frac{(1+g)^t a_5}{\prod_{j=1}^{t}(1+i_j)n_4}\,pm_t + \frac{1}{\prod_{j=1}^{t}(1+i_j)n_4}\,pc_t +$$

$$- \frac{\beta}{\prod_{j=1}^{t}(1+i_j)(1+{}_ti^E_{t+1})n_4}\,{}_tpk^E_{t+1} - \frac{\beta(1+g)^{t+1}(a_1-a_5)}{\prod_{j=1}^{t}(1+i_j)(1+{}_ti^E_{t+1})n_4}\,{}_tpm^E_{t+1} +$$

$$- \frac{\beta}{\prod_{j=1}^{t}(1+i_j)(1+{}_ti_{t+1})n_4}\,{}_tpc^E_{t+1} + \frac{\beta d\alpha_4}{n_4}\ , \tag{4.7}$$

where $n_4 = b_4 + b_6$.

If c_{t-1} in (4.7) is brought from the left to the right hand side and $c_{t-1}-vc_t$ is substituted by c_t-v_{t-1}, (4.7) becomes

$$c_t = \frac{b_6}{n_4}\,v_{t-1} + \frac{(1+g)^t a_5}{\prod_{j=1}^{t}(1+i_j)n_4}\,pm_t - \frac{1}{\prod_{j=1}^{t}(1+i_j)n_4}\,pc_t +$$

$$+ \frac{\beta}{\prod_{j=1}^{t}(1+i_j)(1+{}_ti^E_{t+1})n_4}\,{}_tpk^E_{t+1} + \frac{\beta(1+g)^{t+1}(a_1-a_5)}{\prod_{j=1}^{t}(1+i_j)(1+{}_ti^E_{t+1})n_4}\,{}_tpm^E_{t+1} +$$

$$+ \frac{\beta}{\prod_{j=1}^{t} (1+i_j)(1+{}_t i^E_{t+1}) n_4} \; {}_t pc^E_{t+1} - \frac{\beta d \alpha_4}{n_4} \tag{4.8}$$

By substituting $\prod_{j=1}^{t} (1+i_j)$ by Π_t and the parameter constellations in (4.6) resp. (4.8) by $w_{1,1j}$ ($\geqq 0$), $j = 1,\ldots,8$, resp. $w_{1,2j}$ ($\geqq 0$), $j = 1,\ldots,7$, the reaction equations assume the following form:

$$d_t = - \frac{w_{1,11}}{\Pi_t} pp_t + \frac{w_{1,12}}{\Pi_t(1+{}_t i^E_{t+1})} \; {}_t pk^E_{t+1} + \frac{(1+g)^{t+1} w_{1,13}}{\Pi_t(1+{}_t i^E_{t+1})} \; {}_t pm^E_{t+1} +$$

$$+ \frac{w_{1,14}}{\Pi_t(1+{}_t i^E_{t+1})} \; {}_t pc^E_{t+1} + \frac{w_{1,15}}{\Pi_t(1+{}_t i^E_{t+1})(1+{}_t i^E_{t+2})} \; {}_t pk^E_{t+2} +$$

$$+ \frac{(1+g)^{t+2} w_{1,16}}{\Pi_t(1+{}_t i^E_{t+1})(1+{}_t i^E_{t+2})} \; {}_t pm^E_{t+2} + \frac{w_{1,17}}{\Pi_t(1+{}_t i^E_{t+1})(1+{}_t i^E_{t+2})} \; {}_t pc^E_{t+2} +$$

$$- w_{1,18} \tag{4.9}$$

$$c_t = w_{1,21} v_{t-1} + \frac{(1+g)^t w_{1,22}}{\Pi_t} pm_t - \frac{w_{1,23}}{\Pi_t} pc_t + \frac{w_{1,24}}{\Pi_t(1+{}_t i^E_{t+1})} \; {}_t pk^E_{t+1} +$$

$$+ \frac{(1+g)^{t+1} w_{1,25}}{\Pi_t(1+{}_t i^E_{t+1})} \; {}_t pm^E_{t+1} + \frac{w_{1,26}}{\Pi_t(1+{}_t i^E_{t+1})} \; {}_t pc^E_{t+1} - w_{1,27} \tag{4.10}$$

According to (4.6) the investments in year t are determined by the (deflated) price of heifers in that same period, the (deflated) expected prices for milk and the different kinds of beef and finally a constant. The level of desinvestment also depends on the size of the dairy herd and the inflow of heifers in calf. The weight of each variable is given by a constellation of coefficients from the specification of the revenues and expenditures function. Curious about (4.6) and (4.7) and also about the other equations in (4.3) is that the optimal decisions for year t, apart from the constant and the state variables, only depend on the prices in that same year and the price expectations for the next two periods. Hence, only a part of the future plays a role in these decisions. One would rather suspect these decisions to be gouverned by the price expectations of all years to come before the sale of the farm. It must be admitted that these rules are hardly interpretable in economic terms. Of course, on the basis of the derivation followed, it can not be misunderstood that the equality of marginal revenues and costs hides behind these expressions, but it is as yet not clear to us how to state this in economic terms.

In (4.9) and (4.10) the factors are identified which determine the (optimal) level of the (des)investment in the dairy stock and also the specific influence of each of these variables. By means of these relations one can assess to what extent the stock size reacts on changes in the price for milk.

The average size of the dairy cow stock in period t amounts to

$$\bar{c}_t = \frac{c_{t-1} + c_t}{2} \tag{4.11}$$

The insertion of (4.10) in (4.11) gives this average as a function of amongst others the prices and price expectations in that period. In view of the dependence of c_{t+1} on v_t these prices also influence $\bar{c}_{t+1}$ and $\bar{c}_{t+2}$. Under the assumption that the milk price expectations in (4.9) and (4.10) only depend on pm_t, as far as it concerns milk prices, the effect of a change in the milk price in period t on the average size of the dairy cow stock is expressed by

$$\frac{\partial c_t}{\partial pm_t} + \frac{\partial c_t}{\partial\, {}_t pm^E_{t+1}} \frac{\partial\, {}_t pm^E_{t+1}}{\partial pm_t} + \frac{\partial c_{t+1}}{\partial v_t} \left[\frac{\partial v_t}{\partial\, {}_t pm^E_{t+1}} \frac{\partial\, {}_t pm^E_{t+1}}{\partial pm_t} + \frac{\partial v_t}{\partial\, {}_t pm^E_{t+2}} \frac{\partial\, {}_t pm^E_{t+2}}{\partial pm_t} \right], \tag{4.12}$$

or, shortly, by

$$\frac{\partial c_t}{\partial pm_t} + \frac{\partial c_{t+1}}{\partial v_t} \frac{\partial v_t}{\partial pm_t} \tag{4.13}$$

Under the assumption just mentioned the long run elasticity can be determined by for instance the average of the elasticities in the several years.

$$\frac{\Delta\bar{c}}{\Delta pm}\ \frac{pm}{\bar{c}} = \frac{1}{T} \sum_{t=1}^{T} \left\{ \left[\frac{\partial c_t}{\partial pm_t} + \frac{\partial c_{t+1}}{\partial v_t} . \frac{\partial v_t}{\partial pm_t} \right] \frac{pm_t}{\bar{c}_t} \right\} \tag{4.14}$$

The elements $\frac{\partial c_t}{\partial pm_t}$, $\frac{\partial c_{t+1}}{\partial v_t}$ and $\frac{\partial v_t}{\partial pm_t}$ are obtained by means of the estimates for the corresponding regression coefficients. Should the milk price expectations in (4.9) and (4.10) also depend on other milk prices than the one of period t, then (4.12) has to be adjusted accordingly.

The reaction equations (4.9) and (4.10) are derived at micro level, so the elasticity (4.14) can be estimated using data concerning individual farms. However, if we assume that the same type of model as the one derived holds for all firms in the sector, the conditions for consistent aggregation are satisfied and estimation of (4.14) using data with respect to the sector as a whole is also allowed [4]. Because micro data for but a part of the period considered here are at our disposal, we will estimate the coefficients $\frac{\partial c_t}{\partial pm_t}$, $\frac{\partial c_{t+1}}{\partial v_t}$ and $\frac{\partial v_t}{\partial pm_t}$ using sector data.

Before we can continue with this estimation, we have to find a solution for the following two problems. First, the inflow of heifers in calf is not registered in the Netherlands, so the level of these investments has to be calculated somehow. By regrouping the detailed data of the yearly agricultural May census fifteen "observations" concerning this inflow could be generated. However, in comparison to the number of regressors in (4.9), this number is too small to allow a reliable estimate of the long term elasticity, so reducing the reaction equations turns out to be unavoidable. Second, we do not know the level of the prices for milk, the various kinds of beef and the inflation percentage that the farmers expect for the two years following the decision period. Therefore in the next section we formulate a model allowing the generation of these expectations. After these preparations we can proceed to the estimation of the long term elasticity.

5. The price expectations scheme

In the Community the markets for a number of agricultural products, among them milk and beef, are regulated. The existence of such a regulation without doubt influences the price expectations for the corresponding product. In modelling these expectations it is therefore necessary to incorporate this regulation aspect.

The market regulation for milk and dairy products provides the existence of among other things a guide and a minimum price, derived from the guide price, for milk of a standard quality off farm. The level of these prices is determined every year by the council of the ministers of agriculture of the member states. The level of the guide price is fixed such as is thought to be reasonable for both producers and consumers. The regulation administration tries to realize this level by means of several measures. One of them is taking out of the market the butter and milk powder that do not earn the minimum prices in the market. The price de facto received by the farmers, the market price, is ideally equal to the guide price, but in reality this is seldom the case. More often than not the market price differs from the guide price. However, the administration can not permit itself to let exist a sizable difference to exist for a long time without loosing its credibility.

In view of the preceding we can choose for the milk price in (4.9) and (4.10) the price received by the producer, the market price, or the guide price. For the period considered the market and guide price are highly correlated, so it makes little difference which of them is chosen. However, we prefer the guide price. The motivation for this choice is, that this price in view of the way it comes about, contains information about the development for the market for milk and dairy products which the council considers desirable. For that reason the guide price is a better aid for decisions with respect to the farm than the market price which also experiences the influence of accidental circumstances. Second, by choosing the guide price we incorporate an important aspect of the dairy market regulation in the Community.

Because we take for the milk price of the current year the guide price it is evident to set equal the expected milk prices to the expected guide prices. However, we will only incorporate the expectation for the first year after the decision period. The reason for this is that the information used by the farmer for the expectation for the first year after the decision period is at least for an important part the same as the information he uses for the expectation for the second year after the decision period. As a consequence these expectations are dependent from a statistical point of view and, so one may assume, highly correlated. To avoid this collinearity problem we incorporate only the expectation for the first year. As a result of this reduction the number of regressors in (4.9) and (4.10) decreases, whereas the information, contained in the regressors left out, is, so we suppose, kept. Because these considerations mut.mut. also hold for the expected beef prices, we treat these expectations in the same way. Finally, we suppose that the expected inflation is equal to its last realization,

$$ {}_{t}i^{E}_{t+1} = i_{t} \tag{5.1}$$

Expectations can be modelled in a number of ways. The extremes are the naive scheme on the one hand and the rational expectations hypothesis on the other. In the situation of naive expectations the suppliers of a commodity expect the future price to be equal to the current price, whereas this price expectation in the rational expectations hypothesis depends on all relevant information, not only the current price. For each of these possibilities arguments pro and contra can be given. Generating a naive expectation is an easy job, but more often than not such a scheme can not comply with reality, since only part of the information available to the producers is incorporated in this model. For instance, if we take the current guide price for the expected price, all such information which, as may be assumed, is also relevant for this future price, is in advance discarded as being not relevant. Such information is not only contained in for instance the development of the milk production costs or the average yield, but also in the existence and the size of dairy surpluses on the common market. When the producers repeatedly are informed that the dairy policy is aiming at equality of demand and supply and can suspect that a difference between them will influence prices, one would expect that the producers take into consideration such information in forming their expectations. The insufficient adequacy of the naive scheme in incorporating relevant information can be removed by using the rational expectations model, but changing over to this model would involve a too laborious task, not only since a model of both demand and supply has to be developed, but also because it concerns a regulated market, leaving out still the complications resulting from de- and revaluations by the member states.

To begin, we specify the milk price expectation by a model that, comprising both the guide price of the decision period and a measure for the difference between supply and demand as a correction mechanism, contains elements from both the naive and the rational expectations hypothesis. The deviation between supply and demand can be measured by for instance the expenditures for the common dairy policy or the stocks of butter or skimmed milk powder of the Community. This model is

$$\underline{rp}_t = \delta_{1,1} \frac{1+i_{t-1}}{1+g} rp_{t-1} + \delta_{1,2} x_{t-1} + \underline{\xi}_{1,t}, \tag{5.2}$$

where $\underline{rp}_t$ the guide price in period t, g the procentual growth of the yield per cow, x_{t-1} the measure for the disequilibrium of the common dairy market and $\underline{\xi}_{1,t}$ a stochastic disturbance term having a normal distribution. In (5.2) four factors play a role in the determination of the new guide price (leaving $\underline{\xi}_{1,t}$ a moment out of consideration): the current guide price, the development of the costs of milk production and the productivity and finally a measure for the disequilibrium of the market. According to this model producers to some extent receive a compensation for the increase of production costs, whereas the growth of productivity is partly handed over in the new guide price and so, in principle, in the consumer prices. By the coefficient $\delta_{1,2}$ the development of the guide price and the financial consequences of the market regulation are linked up. Increasing regulation expenditures result cet.par. in pressure on the guide price and so on the market price for the producers, whereas the opposite holds for decreasing expenditures. The term $\underline{\xi}_{1,t}$ finally expresses amongst other things that the determination of the guide price is to some extent the result of package dealing.

The model (5.2) was estimated using as a disequilibrium measure the expenditures for the regulation of the common dairy market in total or per 100 kg. milk, the intervention stocks and several variations on them. None of them showed a significant contribution to the explanation of the variation of the guide price. Only the sign was as might be expected. However, the development of the guide price was very satisfactorily explained by the first regressor $\frac{1+i_{t-1}}{1+g} rp_{t-1}$. Though it is hardly acceptable to find that the guide price is not influenced by the level of the regulation expenditures, think of the reason for the introduction of the co-responsibility levy and the super levy, the investigated specifications did not allow to reach the conclusion of significance of these expenditures.

Because the regressor x_{t-1} did not show a significant contribution, we changed over to the following specification

$$\underline{rp}_t = \delta_{2,1} \frac{1+i_{t-1}}{1+g} rp_{t-1} + \underline{\xi}_{2,t} \tag{5.3}$$

In (5.3) the new guide price is the result of the adjustment of the old guide price for inflation and productivity. For this model the following result was obtained

$$\hat{rp}_t = 1,0093 \frac{1+i_{t-1}}{1+g} rp_{t-1}, \tag{5.4}$$

a result very well in accordance with reality. As the null hypothesis, $\delta_{2,1} = 1$, can not be rejected, we choose the following model for the generation of the milk price expectations

$$_t pm^E_{t+1} = E\{\underline{rp}_{t+1}\} = \frac{1+i_t}{1+g} rp_t \tag{5.5}$$

For the expected beef prices we use a comparable scheme: the expected prices for the different kinds of beef are equal to the present prices, adjusted for inflation.

6. The reduction

As remarked before we dispose of but 15 observations concerning the inflow of heifers in calf. Compared to the number of regressors in (4.9) this is a number too small to allow the calculation of reliable estimates of the coefficients in (4.9) and so of the elasticity. A reduction of the number of regressors is therefore unavoidable. Fortunately, opportunities for such a reduction are available. The first of them is offered by our assumption that the price expectations in the two consecutive years after the decision period are highly correlated. So to avoid collinearity one of these two can be left out of consideration. The second reduction opportunity is offered by the scheme developed for these expectations in the preceding section. The insertion of (5.5) in the term $\frac{(1+g)^{t+1}}{\Pi_t(1 + {}_ti^E_{t+1})} \; {}_tpm^E_{t+1}$ in (4.9) yields the expected (real) milk price to be equal to the current (real) milk price, if we take for ${}_ti^E_{t+1}$ its last realisation, i_t. By a similar reasoning we get a comparable result for the expected beef prices: the expected (real) prices are equal to the current (real) prices. After these two reductions, unavoidably causing loss of part of the information contained in the variables left out, the equations (4.9) and (4.10) assume the following form.

$$v_t = -\frac{w_{3,11}}{\Pi_t} pp_t + \frac{w_{3,12}}{\Pi_t} pk_t + \frac{(1+g)^t w_{3,13}}{\Pi_t} pm_t + $$
$$+ \frac{w_{3,14}}{\Pi_t} pc_t + w_{3,15} \tag{6.1}$$

$$c_t = w_{3,21} v_{t-1} + \frac{(1+g)^t w_{3,22}}{\Pi_t} pm_t - \frac{w_{3,23}}{\Pi_t} pc_t + $$
$$+ \frac{w_{3,24}}{\Pi_t} pk_t + w_{3,25} \tag{6.2}$$

As a consequence of the collinearity it is no longer possible to determine the separate contribution to the elasticity of the current and the expected milk price. Only the sum of these effects can be estimated, given at least the absence of collinearity between the (current) milk price and the other regressors in (6.1) and (6.2). To get an impression of that we examined as to what extent each regressor in (6.1) and (6.2) can be rendered as a linear combination of the other regressors in the corresponding equation.

It turned out that $\frac{pp_t}{\Pi_t}$ and $\frac{pc_t}{\Pi_t}$ in (6.1) and $\frac{pc_t}{\Pi_t}$ in (6.2) can be expressed as an almost perfect linear combination of the other regressors. To a lesser extent this holds for $\frac{pk_t}{\Pi_t}$ too. For that reason a weighted combination of the several beef prices, and not each of them separate, will be used in the regression analyses. As a result of this final reduction round the reaction equations now possess the following form

$$v_t = -w_{4,11} \frac{rvp_t}{\Pi_t} + w_{4,12} \frac{(1+g)^t pm_t}{\Pi_t} + w_{4,13} \tag{6.3}$$

$$c_t = w_{4,21}\, v_{t-1} - w_{4,22} \frac{rvp_t}{\Pi_t} + w_{4,23} \frac{(1+g)^t pm_t}{\Pi_t} + w_{4,24} \tag{6.4}$$

where rvp_t stands for the beef price in period t.
In comparison to (4.9) respectively (4.10) the number of regressors in the equations has been reduced substantially. Of course, this result ought not to be considered as pure profit, as the original specifications have been replaced by considerably less rich ones. It is even questionable, whether a substantial part of the available information has gone lost by this reduction process or not. For that reason not only (6.3) and (6.4) will be estimated, but also variations on it. However, this will not go so far as to include the examination of qua dynamic structure completely different specifications, for instance specifications having lagged endogeneous variables. The reason for that is that we know the decision model on which (6.3) and (6.4) and its variations rest, while such is not the case for specifications having for instance lagged endogeneous variables. So in comparing such different structures we would be restricted to a comparison on but one aspect, the statistical one. However, in such a way no justice is done to the method for obtaining specifications as developed in this paper.

Let us finally remark that the long term elasticity (4.14), based on (6.3) and (6.4) is given by

$$\frac{\frac{\Delta\bar{c}}{\bar{c}}}{\frac{\Delta pm}{pm}} = \frac{1}{T}\sum_{t=1}^{T}\left[\frac{w_{4,23}\frac{(1+g)^t}{\Pi_t} + w_{4,21}\cdot w_{4,12}\frac{(1+g)^t}{\Pi_t}}{\frac{\bar{c}_t}{pm_t}}\right], \tag{6.5}$$

where $w_{4,23}$ and $w_{4,12}$ are positive and $0 < w_{4,21} < 1$, compare (4.6) and (4.8).

7. The estimation

For the estimation of the long term elasticity we take the following models as a starting point

$$\underline{v}_t = -w_{4,11} \frac{rvp_t}{\Pi_t} + w_{4,12} \frac{(1+g)^t pm_t}{\Pi_t} + w_{4,13} + \underline{w}_{1,t}, \tag{7.1}$$

$$\underline{c}_t = w_{4,21}\, v_{t-1} - w_{4,22} \frac{rvp_t}{\Pi_t} + w_{4,23} \frac{(1+g)^t pm_t}{\Pi_t} + w_{4,24} + \underline{w}_{2,t}, \tag{7.2}$$

We suppose that $\underline{w}_{1,t}$ and $\underline{w}_{2,t}$ possess a normal distribution with $E\{\underline{w}_{1,t}\} = E\{\underline{w}_{2,t}\} = 0$. Because the two equations share the person of the decision-maker, the possibility of contemporaneous correlation can not be excluded. So

$$\mathrm{Cov}\{\underline{w}_{1,i}, \underline{w}_{2,j}\} = \delta_{ij} \begin{bmatrix} V\{\underline{w}_1\} & \mathrm{Cov}\{\underline{w}_1, \underline{w}_2\} \\ \mathrm{Cov}\{\underline{w}_2, \underline{w}_1\} & V\{\underline{w}_2\} \end{bmatrix},$$

where $\delta_{ij} = 1$ for $i = j$ and 0 elsewhere, $i,j = 1,2,\ldots$ However, in view of the reduction rounds above and also for the sake of completeness we will first deal with the case, where

$$\mathrm{Cov}\{\underline{w}_{1,t}, \underline{w}_{2,t}\} = 0.$$

In judging the quality of the estimation results for (7.1), (7.2) and its variations we will be guided by both economic and statistical considerations. First of all, a significant part of the variation of the dependent variable should be explained. Also the regression coefficients should have the expected sign or lie within the expected range. In observing a high degree of collinearity we will in general not adhere to the original specification, but change it by dropping one or more explanatory variables, unless this causes a substantial change of the original specification. In that case we prefer to adhere to the original specification. Such an approach may be hardly defendable, when a specification is obtained via ad hoc reasoning, in a situation as the present one, where the specifications are well founded, matters are more intricate. Neutralizing collinearity via a change in the specification would here imply that the decision model underlying this specification would be abandoned partly. When this change concerns a minor aspect, like the number of lagged exogeneous variables cancelling the corresponding variable may form an acceptable solution for the difficulties evoked by collinearity. However, when it concerns an important aspect as for instance the dynamics of the endogeneous variables or the linearity of the decision rules, then observing a high degree of collinearity calls for a critical re-examination of the model, but in our opinion forms insufficient reason to discard the model partly. As a consequence of this approach the regression coefficients may have a relatively large variance and may be sensitive to adding or dropping observations. As a further consequence the significance of these coefficients, though highly desirable, can not be handled as decisive.

In what follows, first some of the OLS results for (7.1) and its variations will be dealt with, then those for (7.2) and finally the results for $\underline{v}_t$ and $\underline{c}_t$ together.

Table 7.1 The OLS estimates for (7.1)

$$\hat{v}_t = \underset{(-4{,}334)}{-75.585} \frac{rvp_t}{\Pi_t} + \underset{(4{,}394)}{17.551{,}3} \frac{(1+g)^t pm_t}{\Pi_t} + \underset{(1{,}990)}{343.669.4}$$

$n = 15$ $\quad F = 17{,}281$ $\quad \bar{R}^2 = 0{,}699$ $\quad \hat{\sigma}(w) = 48.510{,}4$

$DW = 1{,}325$ $\quad \hat{\rho} = 0{,}310$ $\quad Con = 31{,}51$

In this table n stands for the number of observations, F for the value of the test statistic $\underline{F}$, $\hat{\sigma}(w)$ for the estimated standard deviation of the residuals, $\bar{R}^2$ for the adjusted multiple correlation coefficient, DW for the Durbin-Watson test statistic, $\hat{\rho}$ for the estimated first order autocorrelation and Con for the condition number. The condition number measures the degree of collinearity. Following Belsley a.o. [5] we associate weak dependency with condition indexes around 5 or 10, whereas moderate to strong relations are associated with condition indexes of 30 to 100. The numbers in parentheses finally give the value of the test statistic $\underline{t}$.

Having a F value of more than 17 the model (7.1) provides a good explanation of the development of the number of heifers in calf. Also the price variables have the expected sign and are significant. However, the constant is not significant. Further, no conclusion can be reached about the null hypothesis $\rho = 0$.

Now, as remarked above, one can imagine that as a consequence of the reduction rounds before, variables that should have been incorporated, are wrongly left out of (7.1). In view of that some (linear) specifications having a greater or smaller number of explanatory variables were examined. The most adequate model within this group turns out to be the following specification:

$$\underline{v}_t = -w_{5,11}\frac{rvp_t}{\Pi_t} + w_{5,12}\frac{(1+g)^{t-1}pm_{t-1}}{\Pi_{t-1}} + w_{5,13}\frac{(1+g)^t pm_t}{\Pi_t} + \underline{w}_{3,t}, \tag{7.1.a}$$

for which the results are given in table 7.1.a.

Table 7.1.a The OLS results for (7.1.a)

$$\hat{v}_t = \underset{(-4,304)}{-63.821,7}\frac{rvp_t}{\Pi_t} + \underset{(3,034)}{10.841,8}\frac{(1+g)^{t-1}pm_{t-1}}{\Pi_{t-1}} + \underset{(3,577)}{14.014,9}\frac{(1+g)^t pm_t}{\Pi_t}$$

$n = 14$ $\hat{\sigma}(w) = 43.016,04$ Con $= 31,15$

The F and $\bar{R}^2$ values for this model are omitted, because they can not be compared to those of the model (7.1), differently defined as these statistics are for models with and without a constant.

Having the same number of regressors as (7.1) and nearly the same condition index, this model shows a considerably lower residual variance. Judging from this residual variance (7.1.a) contains more information concerning $\underline{v}_t$ than (7.1). For that reason this specification with its dynamics in the exogeneous variables may be considered to gather the prices and price expectations in (4.9) more adequately than the static specification (7.1).

Let us now consider the model for the development of the dairy cow stock, (7.2). Table 7.2 gives the results of the OLS procedure.

Table 7.2 The OLS results for (7.2)

$$\hat{c}_t = \underset{(1,390)}{0,9745\, v_{t-1}} - \underset{(-1,053)}{71.469,3\, \frac{rvp_t}{\Pi_t}} + \underset{(1,391)}{19.763,8\, \frac{(1+g)^t pm_t}{\Pi_t}} + \underset{(3,059)}{1.192.140,7}$$

$n = 14$ $F = 17,87$ $\bar{R}^2 = 0,796$ $\hat{\sigma}(w) = 79.213,3$

$DW = 1,056$ $\hat{\rho} = 0,243$ Con $= 76,34$

As can be observed in table 7.2, the specification (7.2) clearly suffers from collinearity: having a F value of nearly 18 it combines a strong explanatory power with the non-significance of three out of four regressors. However, it must be admitted that the coefficients have their expected sign, while the v_{t-1} coefficient lies between 0 and 1. No conclusion is reached about the null hypothese $\rho = 0$.

Because the collinearity affects the reliability and stability of the estimates, we searched to overcome this difficulty somehow. The first opportunity partly to get rid of the collinearity is therefore offered by leaving out v_{t-1}. Because in such a way an in our opinion important feature of this specification would be discarded, this possibility is left out of consideration. When this last model nevertheless is estimated, the hypothesis of zero autocorrelation has to be rejected. That might be caused by chance or by the circumstance, that an important variable wrongly has not been explicitly incorporated in the model for $\underline{c}_t$. Substitution of v_{t-1} in (7.2) by the model (7.1.a) offers the second chance. This last model provides a good explanation of the development of the number of heifers in calf, so the information contained in v_{t-1} with respect to $\underline{c}_t$ will be preserved partly. Because this substitution deteriorates the ratio between the number of observations and regressors, and the condition index sizably increases, also this approach does not form an acceptable outlet. The three remaining possibilities, either $\frac{rvp_t}{\Pi_t}$ or the constant or both of them are cancelled, neither prove to form an, also for the rest, acceptable solution for this difficulty.

Now that the high degree of collinearity can not be decreased by means of a further reduction of the number of regressors, and the estimation results do not oppose to presume that the variable v_{t-1} should be explicitly included in the model for $\underline{c}_t$, the specification (7.2) can be considered adequately to cover the specification (4.10) in spite of its adhering difficulties.

After this first reconnaissance of the equations (7.1) and (7.2) separately, we proceed with the simultaneous estimation of these specifications.

To begin, the GLS procedure was applied to the original models for $\underline{v}_t$ and $\underline{c}_t$, (7.1) and (7.2). Though this procedure brings about shifts in the estimates for the regression coefficients, the same conclusions can be drawn as formulated before for each of these two models separately. The explanation for it is that the residual terms are but weakly correlated. This also proves to hold for the case, where the GLS procedure is applied to other combinations (of variations) of $\underline{v}_t$ and $\underline{c}_t$. For that reason it was decided to examine not all of the possible combinations, but to confine ourselves to a final estimation of the two specifications that have been selected before as the most adequate ones, that means the model (7.1.a) and the model (7.2). The results are shown in table 7.3.

Table 7.3 The GLS results for the variations selected in the OLS stage

$$\hat{v}_t = \underset{(-4{,}302)}{-63.785{,}4}\ \frac{rvp_t}{\Pi_t} + \underset{(2{,}799)}{9.916{,}9}\ \frac{(1+g)^{t-1} pm_{t-1}}{\Pi_{t-1}} + \underset{(3{,}835)}{14.920{,}6}\ \frac{(1+g)^{t} pm_t}{\Pi_t}$$

$$\hat{c}_t = \underset{(1{,}165)}{0{,}8055}\ v_{t-1} - \underset{(1{,}270)}{85.148{,}2}\ \frac{rvp_t}{\Pi_t} + \underset{(1{,}667)}{23.451{,}6}\ \frac{(1+g)^{t} pm_t}{\Pi_t} +$$

$$+ \underset{(3{,}188)}{1.223.294{,}2}$$

The comparison of the results in table 7.3 with those in the tables 7.1.a and 7.2 learns that the GLS and OLS procedure yield almost identical estimates for $\underline{v}_t$, whereas the GLS method finds a lower effect on $\underline{c}_{t-1}$ of v_{t-1} and a higher of the other regressors. An investigation into the reliability and stability of the GLS estimates had to be omitted due to the small number of observations.

On the basis of the model in the table 7.3 the long term elasticity of milk supply with respect to the milk prices is after adjustment via $\frac{\partial v_t}{\partial z_t} \cdot \frac{\partial z_t}{\partial pm_t}$, where $z_t = \frac{(1+g)^t pm_t}{\Pi_t}$, given by

$$\frac{\frac{\Delta \bar{c}}{\bar{c}}}{\frac{\Delta pm}{pm}} = \frac{1}{T} \sum_{t=1}^{T} \left\{ \frac{\left[\frac{\partial c_t}{\partial z_t} + \frac{\partial c_{t+1}}{\partial v_t} \cdot \frac{\partial v_t}{\partial z_t} + \frac{\partial c_{t+2}}{\partial v_{t+1}} \frac{\partial v_{t+1}}{\partial z_t} \right] \frac{(1+g)^t}{\Pi_t}}{\frac{\bar{c}_t}{pm_t}} \right\}, \tag{7.3}$$

because the effect of a milk price change stretches over a three years period. In the first year the level of culling is influenced and in the next two years the inflow of the heifers in calf is affected.
Using the estimates for $\frac{\partial c_t}{\partial z_t}$, $\frac{\partial c_{t+1}}{\partial v_t} \left(= \frac{\partial c_{t+2}}{\partial v_{t+1}} \right)$, $\frac{\partial v_t}{\partial z_t}$ and $\frac{\partial v_{t+1}}{\partial z_t}$ included in table 7.3 the long term elasticity turns out to be

$$\frac{\frac{\Delta \bar{c}}{\bar{c}}}{\frac{\Delta pm}{pm}} = \frac{\{23.451,6 + 0,8055 \;.\; 14.920,6 + 0,8055 \;.\; 9.916,9\}}{14} \cdot \frac{2,465287}{10.000}$$

$$= 0,765 \tag{7.4}$$

Having a value of 0,765 this elasticity lies considerably below the turning point between an elastic and inelastic reaction. When a confidence interval is constructed around this point estimation, it stays well beneath this point. So, during the period 1969-1984 a milk price change of 1 percent brought about the size of the dairy herd (and cet.par. the level of the milk supply) to change by about 0,75 percent.

8. Conclusion

In this paper the long term elasticity of the milk supply with respect to the milk price in the Netherlands during the period 1969-1984 is estimated. During this period the same uniform regime applied to all dairy farmers and they were free to choose whatever quantity of milk they wanted to supply, in contrast to the years after 1984. This elasticity was estimated on the base of a relation that was not postulated, but instead derived from an optimization model. This model concerns the decision problems with respect to the size and composition of the live stock the farmers are confronted with. The solution of this model provides a relation between the (optimal) level of the (des)investment in the dairy cow stock - and hence cet.par. the level of the milk supply - and the milk price. This specification forms a starting point for estimating to what extent the milk supply reacts on changes in the prices for milk in the long run. This result has been reached on the basis of several simplifying assumptions. The most important simplification has been that we restricted us to the case where restrictions such as (3.5) (or comparable restrictions with respect to labour, dead stock and capital) are not active. Answering the question, whether and if so under what conditions alpha-numerically specified decision rules (and so reaction equations) can be obtained for the situation where such restrictions are active, would therefore be a desirable continuation of this study. Apart from its evident contribution to enlarging the reality content of the model, such an extension could bring within reach a well founded starting point for an investigation into the production and investment behavior in the situation of production rationing.

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Appendix

The data

Year	v_t [1)]	c_t [2)]	pk_t [2)]	pp_t [2)]	pc_t [2)]	rvp_t [3)]	pm_t [2)]	Π_t [2)]
1969-70			7,16	4,94	4,44	5,02	35,30	0,9651
1970-71	597.848	1.967.149	7,91	4,78	4,31	5,03	35,88	1
1971-72	564.530	1.975.921	9,35	5,25	4,85	5,70	38,04	0,9923
1972-73	574.719	2.039.957	12,07	6,01	5,40	6,67	40,44	0,9980
1973-74	600.299	2.174.530	11,00	5,99	5,29	6,42	41,99	1,2040
1974-75	689.365	2.254.899	8,20	5,68	5,13	5,77	45,43	1,3850
1975-76	642.464	2.258.771	9,84	6,16	5,71	6,52	50,86	1,4705
1976-77	618.997	2.282.510	11,10	6,55	5,99	6,99	54,51	1,6937
1977-78	632.416	2.245.058	11,95	6,94	6,27	7,39	56,97	1,5887
1978-79	683.518	2.295.416	13,13	7,00	6,29	7,62	58,05	1,5417
1979-80	711.313	2.368.963	12,29	6,88	6,18	7,38	58,79	1,8354
1980-81	769.725	2.399.593	11,15	7,06	6,38	7,35	61,40	1,7521
1981-82	801.061	2.419.048	12,33	7,69	7,01	8,07	66,49	1,8420
1982-83	807.133	2.470.899	13,43	8,07	7,30	8,52	70,68	1,8940
1983-84	805.333	2.557.234	12,89	7,87	6,92	8,15	72,84	2,0277
1984-85	776.865	2.583.741						

1) Estimated using the yearly agricultural May census data.

2) Landbouwcijfers (Agricultural data), LEI/CBS (Agricultural Economics Research Institute/Netherlands Central Bureau of Statistics).
The prices are based on the agricultural year, not on the calendar year.

3) rvp_t = 0,17 pk_t + 0,23 pp_t + 0,60 pc_t.

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